where:

$$f_1(r) = \left(\frac{r_2}{r}\right)^2 \log k_2 + k_2^2 \log\left(\frac{r}{r_2}\right) + \log\left(\frac{r_1}{r}\right)$$

$$f_2(r) = -\left(\frac{r_2}{r}\right) \log k_2 + k_2^2 \log\left(\frac{r}{r_2}\right) + \log\left(\frac{r_1}{r}\right) + k_2^2 - 1$$

$$f_3(r) = -4(1+\nu)\left(\frac{r_2}{r}\right) \log k_2 + 4(1-\nu)\left[k_2^2 \log\left(\frac{r}{r_2}\right) - \log\left(\frac{r}{r_1}\right)\right] - 4(k_2^2 - 1) \tag{23a-c}$$

and where $(\sigma_r)_c$, $(\sigma_\theta)_c$, and $(u)_c$ are given by Equations (16a-c) and (17a, b) for $k_n = k_2$, $p_{n-1} = p_1$, $p_n = p_2$, and $E_n = E_2$. For a ring segment p_1 and p_2 are related for equilibrium as follows:

$$p_2 = p_1/k_2 \tag{24}$$

Formulas for the constants β_1, G_1, and M_1 (functions of k_2) are given in Appendix A. M_1 represents a bending moment that causes a bending displacement v as shown in Equation (22b).

Pin Segment

The solution for the pin segment is more complicated due to the pin loading at r_2. The resulting expressions are:

$$\sigma_r = (\sigma_r)_c + \frac{4M_2p_1}{\beta_1} f_1(r) + g_{m1}(r)\cos m\theta$$

$$\sigma_\theta = (\sigma_\theta)_c + \frac{4M_2p_1}{\beta_1} f_2(r) + g_{m2}(r)\cos m_\theta \tag{25a-c}$$

$$\tau_{r\theta} = g_{m3}(r)\sin m\theta$$

$$\frac{u}{r} = (u)_c + \frac{M_2p_1}{E_2\beta_1} f_3(r) + \frac{G_2p_1}{r}\cos\theta + \frac{1}{E_2} g_{m4}(r)\cos m\theta \tag{26a, b}$$

$$\frac{v}{r} = \frac{8M_2p_1}{E_2\beta_1}(k_2^2 - 1)\theta - \frac{G_2p_1}{r}\sin\theta + \frac{1}{E_2} g_{m5}(r)\sin m\theta$$

where $(\sigma_r)_c$, $(\sigma_\theta)_c$, and $(u)_c$ are again given by Equations (16a-c) and (17a, b) for $k_n = k_2$, $p_{n-1} = p_1$, $p_n = p_2$, and $E_n = E_2$. For a pin segment p_2 is related to p_1 as follows:

$$p_2 = \frac{(m^2-1)(1 + 2\cos\pi/m)}{2(m^2-2)(1 + \cos\pi/m)} \quad (\frac{p_1}{k_2}) \tag{27}$$

where m defined as

$$m = 2N_s \tag{28}$$

and where N_s is the number of segments per disc.

The functions $f_1(r)$, $f_2(r)$, and $f_3(r)$ are again given by Equations (23a-c) and β_1, G_2, M_2, g_{m1}, ..., $g_{m5}(r)$ are given in Appendix A.

The elasticity solutions now can be used to determine formulas for maximum pressure capability from the fatigue relations. This is done in the next section.

NONDIMENSIONAL PARAMETER ANALYSIS

The maximum pressure that is possible in any one container is a function of the material fatigue strength, the amount of prestress, the number of components N, and the wall ratios k_n. In order to determine the function dependence on these variables and to determine the best designs a nondimensional analysis is now presented. The calculations for the analysis of each design were programmed on Battelle's CDC 3400 computer.

Multi-Ring Container

Static Shear Strength Analysis

Although a fatigue criterion of failure has been chosen it is illustrative to review an analysis based upon static shear strength for ductile materials first conducted by Manning(4). The method outlined here differs from that of Manning and is more straightforward. In this analysis the optimum design is found such that each component of the same material has the same value of maximum shear stress S under the pressure load p. The given information is $p_o = p$, $p_N = 0$, and K. The unknowns are the interface pressures p_n, (N-1) in number; the k_n, N in number and S. The total unknowns are 2N. There are N equations resulting from Equation (18) and having the form

$$S = (p_{n-1} - p_n)\frac{k_n^2}{k_n^2-1}, \; n = 1, 2, \ldots, N \tag{29}$$

There is Equation (7) relating the k_n and K. Also N-1 equations can be formulated from the requirement that S be a minimum, i.e.,